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ORIGINAL RESEARCH

An efficient route design for solid waste collection using graph theory and the algorithm of the traveling agent in dynamic programming

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Abstract In Santo Domingo de los Tsa'chilas province, Ecuador, the population grows proportionally to the territorial extension in urban and rural parishes; therefore, the conception of domestic solid waste has increased exponentially. In this context, in recent years, the distribution of routes for waste collection has not been dealt with or technically explored. The research objective is to apply the theory of graphs to the sector and use the exact method of the Travel Agent Problem (TSP) in dynamic programming to generate optimal routes by sectors. In addition to measuring the variables longitudinally, we test the researcher's hypothesis using parametric techniques for independent samples in the variable's travel time and distance between the usual route and the new route in the Río Verde parish of Santo Domingo Canton.

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Introduction

Santo Domingo de los Colorados city presents an accelerated expansion due to its geographical location, immigration, extensive commercial activity, and the offer of lots in large areas of land. Favor urban growth in this sector, as the population grows, and strengthening its commercial, industrial, and agricultural activities, increases solid waste generation. Population growth in the urban area and land occupation availability is one of the main factors for increasing the production of garbage and the deposit of this in wrong places.

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The ECOAMBIENTAL company collects solid waste in the Río Verde parish in the Santo Domingo de los Colorados canton; this company won the public tender to hire this service. This activity has been carried out since May 23, 2011, and ends on November 11, 2018, with a per capita production of 0.91 kg/day. (Venegas, 2018).

That is why we now want to implement a new distribution of stable waste collection truck routes applying graph theory and the peddler's problem in dynamic programming in Python considering the variables time and distance between nodes (Abbatecola et al., 2016)

> There is a set of nodes together with the cost of the route between each pair of nodes; the problem of the street vendor, or TSP for short, is to find the cheapest route to visit all the points and return to the initial node. (Applegate & Bixby, 2007)
> The TSP is to find the best possible way to visit all the cities and return to the starting point, having a set of cities and the cost of the trip (or distance) between each possible pair. (Matai et al., 2010)

First, we are looking for a closed route through all the places we want to visit; this means that the route will end where it started. Second, each place is visited precisely once on our route. Since all lengths are positive, visiting a place twice would not give us a better result. As each place is visited exactly once, and there is an infinite number of possible routes through each place. We call this possible to go through a solution for the TSP, as there are many solutions. We know that there is a route with minimum travel costs, and we call it an optimal solution if there is no other solution with a strictly lower travel cost.

As a general objective in our research, we propose to implement new routes for solid waste collection in cooperatives, sectors, and urbanizations, which allows obtaining a considerable reduction in the time and distance variables in the provision of this essential service in the Río Verde parish, Santo Domingo de los Colorados city.

While as specific objectives, we will:

- Design the Bellman-Held-Karp algorithm in Python language using dynamic programming
- Exemplify the gathering of information using a distance matrix of each node to travel in the San Ignacio Location of the Río Verde parish
- Obtain the new routes for the collection of household solid waste in the Río Verde parish
- Evaluate the system with a pilot route at the "16 de Marzo" neighborhood within the Río Verde parish

The research hypothesis is:

> The new solid waste collection routes in the Río Verde parish produce significant savings in time and distance of the collection truck.

Methodology

The research work reaches an application level. It seeks to find the best route for solid waste collection trucks; it is also longitudinal because when testing the hypothesis, we take several measures comparing the new route with the old route. We apply the quantitative method because we gather information through deductive analysis and interpretation of AutoCAD software information to design new routes applying graph theory and the final hypothesis contrast with SPSS.

Bibliographic, field, and descriptive research was also applied because we take bibliographic sources to conceptualize and deepen judgments issued by different authors in solution edges for the Problem of the Traveling Agent; in turn, we collect information in the field (time and distance between nodes) of the routes followed by the solid household waste collection trucks within the Río Verde parish.

Modeling the problem of the traveling agent in a graph with 4 nodes

A graph $G = (V;E)$ is a pair of sets V and E where V is a set of vertices and E is a set of edges. Each element $e = \{v, u\} \in E$ is an edge exactly between two vertices v, u in V.

We have that $V = \{1, 2, 3, 4\}$ and $E = \{\{1, 2\};\{1, 3\};\{2, 3\};\{2, 4\};\{3, 4\}\}$, we represent it in Fig. 1.

For $v, u \in V$, we say that u is a neighbor of v if $\{v, u\} \in E$. We denote $N(v)$ as the set of all neighbors of v.

For each point, we make a vertex that belongs to V. Whenever it is possible to travel from point v to point u directly, we make the edge $e = \{v, u\} \in E$. Now, we have a graph $G = (V;E)$ that it represents the space that

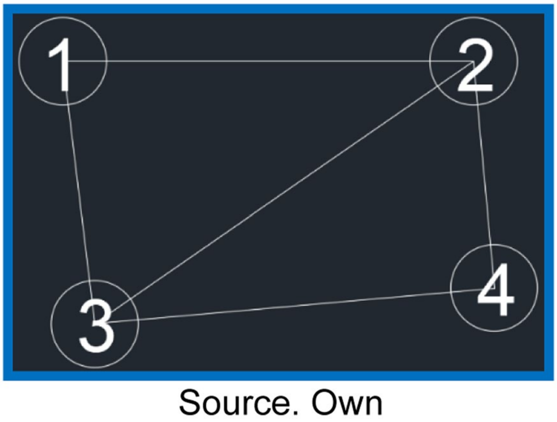


Fig. 1 Graph with 4 nodes and 5 edges

our seller can travel. Now, we propose the weight or distance to quantify the distance between each point. A weight function or a distance function in a graph $G = (V;E)$ is a function $w : E \rightarrow R$ that gives each edge of E a real number. Now, the travel costs to get from v to u are denoted by $w(v;u)$. We use the edges with constants of Q to guarantee an optimal and well-defined value. We will assume that for all TSP instances, we will have $w(e) > 0 \forall e \in E$. Now, suppose we want to measure the travel costs (on time) between v and u by car. The geometric distance in kilometers is a simple measure, but not necessarily the most accurate. Taking the length of a path from v to u can be a more precise measure. Since not all roads have the same speed limit, a shorter route does not always represent a faster route. Before continuing, we take a look at some definitions that help us identify the characteristics of a graph.

For graph $G = (V; E)$, we have to:

$(v_{1,}v_2 \ldots ..v_{k-1}, v_{k,})$ is a path from G that goes from v_1 to v_k if $\{v_{i,}v_{i+1}\} \in E, \forall 1 \leq i \leq k-1$.

A path is as such if it is only used once.

$(v_{1,}v_2 \ldots ..v_{k-1}, v_{k,})$ is a cycle or a circuit of G if $v_1 = v_k$, $\{v_{i,}v_{i+1}\} \in E$ and $v_1 \neq v_j \forall 1 \leq i \leq k-1$, $i \neq j$.

A path or cycle of G is Hamiltonian if it contains all the vertices of V.

A graph G is Hamiltonian if there is a Hamiltonian cycle of.

For graph $G = (V; E)$, we have to (Fig. 2):

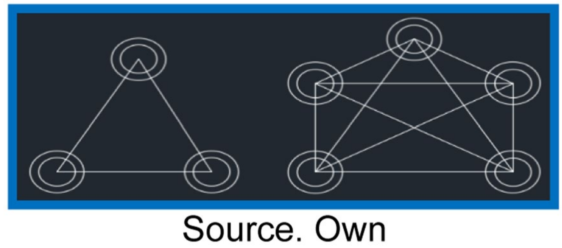


Fig. 2. Two graphs with 3 and 5 nodes respectively

G is connected if there is a path between each pair of vertices.

G is complete if there is a border between each pair of vertices. Note that a complete (unweighted) graph with n vertices is unique. We denote K_n for the complete graph with $|V| = n$.

For $S \subseteq V$, we denote $\delta(S)$ as the set of edges that have exactly one final vertex in S.

We denote $d(v)$ as the degree (number of adjacent edges) of a vertex $\in V$.

We denote $\delta(G)$ as the minimum grade of G.

Now, we can define the problem of the traveling salesman, and we will define it in the following way: given a graph G and a function of weight w on G, determine the minimum weight of the Hamiltonian cycle in G. The weight of the cycle is the sum of all weights of the edges in the cycle. To find a Hamiltonian cycle of minimum weight, we will say that G is Hamiltonian; otherwise, there is no such route. To know if a graph is Hamiltonian can sometimes be quite difficult. There are several ways to discover that a graph is Hamiltonian or not, but in many cases, these methods will leave the question uncertain.

Survey of information in the distance matrix of the San Ignacio place in the Río Verde parish

There are about 100,000 inhabitants in the Río Verde parish (Figs. 3 and 4). We will distribute them in residential neighborhoods, citadels, and urbanizations, to collect information using graphs: Ciudadela del Chofer, Urb. Los Girasoles, Coop. Patria Nueva, Coop. Unión Cívica, Coop. Alejandro Montesdeoca, Coop. Las Macadamias, Coop. Nuevo Amanecer, Coop. Unión Santodominugueña, Coop. Pueblo en Marcha, Coop. Defensores de Paquisha, Coop.

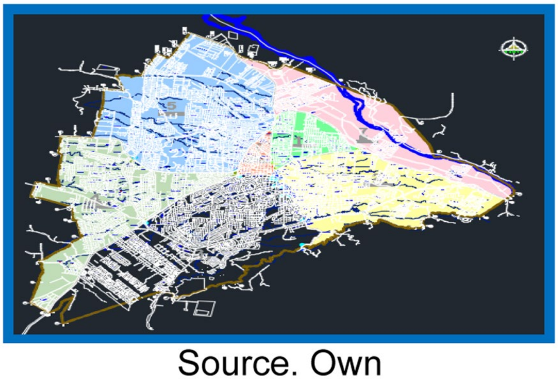


Fig. 3 Santo Domingo city map

Pueblo unido Venceremos, Coop. Santa Martha sectores 1, 2, 3, 4, 5, 6, 7, Coop. Juan Montalvo, Coop. 20 de Octubre, Coop. Rumiñahui, Coop. Asistencia Municiapal, Coop. Nueva República, Urb. Chanchay, Ubr. Los Faisanes, Coop. Aquepi, Urb. Baltra, Urb. Los Pambiles, Urb. Ierac 53, Urb. Jardines del Colorados, Urb. Las Palmeras, Coop. Liberación Popular, Urb. Skinner.

In the San Ignacio cooperative (Fig. 5), there are 14 points; using AutoCad, we draw the graph on a 1: 1 scale.

We draw the 14 points where the waste collection truck will pass; the time and distance from point 1 to 14 will be optimized and returning to the starting point (Table 1).

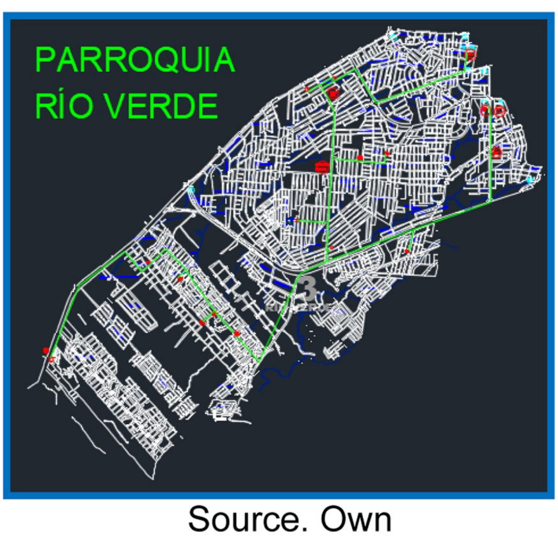


Fig. 4 Map of the Rio Verde parish

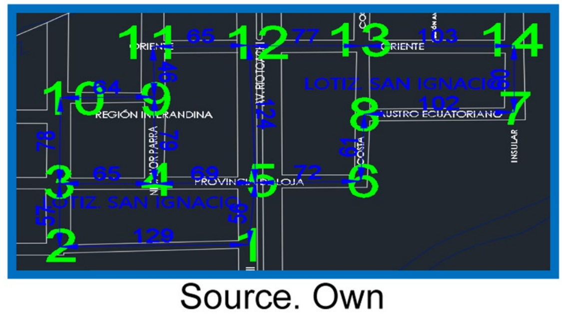


Fig. 5 San Ignacio map

Exemplification of the Traveler Agent Problem method with 4 nodes manually

Exemplify by taking cities $1, 2, ..., N$ and suppose that we begin in city 1, and the distance between city i and city j is (di,j). Consider the subsets $S \subseteq \{2, ..., N\}$ of cities and, for $c \in S, seaD(S,c)$ be the minimum distance, beginning in city 1, visiting all cities in S and ending in the city c.

First phase: if $S = \{c\}$, then $D(S,c) = d_{1,c}$. Otherwise:

Second phase: the minimum distance for a complete tour of all cities is $M = min_{c \in \{2,......,N\}}(D(\{2, ..., N\}, c) + d_{c,1})$

A path $n_1, ..., n_N$ is the minimum distance just when it satisfies $M = D(\{2, ..., N\}, n_N) + d_{n_{N,1}}$.

From the four nodes (Fig. 6), we will raise a symmetric distance matrix, where we will apply the manual mathematical calculations, where we also show the diagonal of zeros (Table 2).

Description of functions:

g (x,S): from 1, the minimum path cost ends at vertex x, passing the vertices in set S exactly once.

c_xy– the cost of the edge ends in x from y.

p (x,S): the second to the last vertex of x in the set S. It was used to build the TSP route at the end.

$k = 0$, null set:

Set $\varnothing$:

$$\mathbf{g}\ (2, \varnothing) = \mathbf{c}_{21} = 1293$$

$$\mathbf{g}\ (3, \varnothing) = \mathbf{c}_{31} = 713$$

Table 1 Distance matrix

Distance matrix San Ignacio place in meters (m) with 14 nodes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	129	186	125	56	128	291	189	204	264	245	180	257	360
2	129	0	57	122	185	257	359	318	201	135	247	309	386	489
3	186	57	0	65	134	206	369	267	144	78	188	253	330	433
4	125	122	65	0	69	141	304	202	79	143	125	190	186	289
5	56	185	134	69	0	72	235	133	148	212	189	124	201	304
6	128	257	206	141	72	0	163	61	220	284	266	331	408	511
7	291	359	369	304	235	163	0	102	351	415	305	240	163	60
8	189	318	267	202	133	61	102	0	281	345	327	257	334	437
9	204	201	144	79	148	220	351	281	0	64	46	111	178	281
10	264	135	78	143	212	284	415	345	64	0	110	175	252	355
11	245	247	188	125	189	266	305	327	46	110	0	65	142	245
12	180	309	253	190	124	331	240	257	111	175	65	0	77	180
13	257	386	330	186	201	408	163	334	178	252	142	77	0	103
14	360	489	433	289	304	511	60	437	281	355	245	180	103	0

Source: Own

$$\mathbf{g}\,(4, \varnothing) = \mathbf{c}_{41} = 821$$

$k = 1$, consider single element sets:

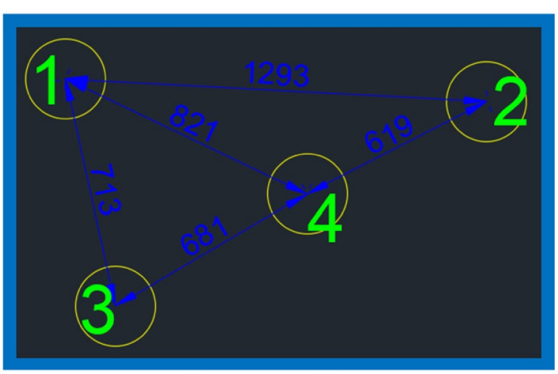
Set {2}:

$$\mathbf{g}\,(3, \{2\}) = \mathbf{c}_{32} + \mathbf{g}\,(2, \varnothing) = \mathbf{c}_{32} + \mathbf{c}_{21} = 1300 + 1293 = 2593;\ \mathbf{p}\,(3, \{2\}) = 2$$

$$\mathbf{g}\,(4, \{2\}) = \mathbf{c}_{42} + \mathbf{g}\,(2,) = \mathbf{c}_{42} + \mathbf{c}_{21} = 619 + 1293 = 1912;\ \mathbf{p}\,(4, \{2\}) = 2$$

Set {3}:

$$\mathbf{g}\,(2, \{3\}) = \mathbf{c}_{23} + \mathbf{g}\,(3,) = \mathbf{c}_{23} + \mathbf{c}_{31} = 1300 + 713 = 2013;\ \mathbf{p}\,(2, \{3\}) = 3$$



Source. Own

Fig. 6. Example with 4 nodes

$$\mathbf{g}\,(4, \{3\}) = \mathbf{c}_{43} + \mathbf{g}\,(3,) = \mathbf{c}_{43} + \mathbf{c}_{31} = 681 + 713 = 1394;\ \mathbf{p}\,(4, \{3\}) = 3$$

Set {4}:

$$\mathbf{g}\,(2, \{4\}) = \mathbf{c}_{24} + \mathbf{g}\,(4,) = \mathbf{c}_{24} + \mathbf{c}_{41} = 619 + 821 = 1440;\ \mathbf{p}\,(2, \{4\}) = 4$$

$$\mathbf{g}\,(3, \{4\}) = \mathbf{c}_{34} + \mathbf{g}\,(4,) = \mathbf{c}_{34} + \mathbf{c}_{41} = 681 + 821 = 1502;\ \mathbf{p}\,(3, \{4\}) = 4$$

$k = 2$, consider sets of 2 elements:

Set {2,3}:

$$\begin{aligned}\mathbf{g}\,(4, \{2, 3\}) &= \mathbf{min}\{\mathbf{c}_{42} + \mathbf{g}\,(2, \{3\}); \mathbf{c}_{43} + \mathbf{g}\,(3, \{2\})\} \\ &= \mathbf{min}\{619 + 2013; 681 + 2593\} \\ &= \mathbf{min}\{2632; 3274\} = 2632\end{aligned}$$

$$\mathbf{p}\,(4, \{2, 3\}) = 3$$

Set {2,4}:

Table 2 Example matrix with 4 nodes

	1	2	3	4
1	0	1293	713	821
2	1293	0	1300	619
3	713	1300	0	681
4	821	619	681	0

Source: Own

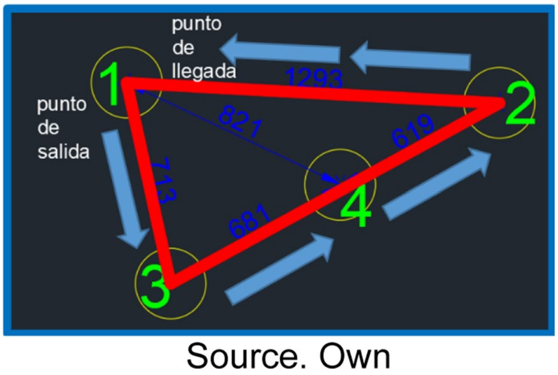


Fig. 7. Path of the example with 4 nodes

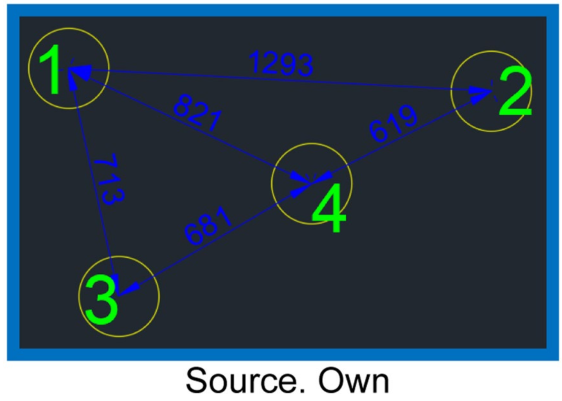


Fig. 8. Example with 4 nodes applying the programming code

$$\mathbf{g}\,(3,\{2,4\}) = \mathbf{min}\{\mathbf{c}_{32} + \mathbf{g}\,(2,\{4\});\mathbf{c}_{34} + \mathbf{g}\,(4,\{2\})\}$$
$$= \mathbf{min}\{1300 + 1440;681 + 1912\}$$
$$= \mathbf{min}\{2740;2593\} = 2593$$

$$\mathbf{p}\,(3,\{2,4\}) = 4$$

Set {3,4}:

$$\mathbf{g}\,(2,\{3,4\}) = \mathbf{min}\{\mathbf{c}_{23} + \mathbf{g}\,(3,\{4\});\mathbf{c}_{24} + \mathbf{g}\,(4,\{3\})\}$$
$$= \mathbf{min}\{1300 + 1502;619 + 1394\}$$
$$= \mathbf{min}\{2802;2013\} = 2013$$

$$\mathbf{p}\,(2,\{3,4\}) = 3$$

Duration of an optimal tour:

$$\mathbf{f} = \mathbf{g}\,(1,\{2,3,4\}) = \mathbf{min}\{\mathbf{c}_{12} + \mathbf{g}\,(2,\{3,4\}), \mathbf{c}_{13} + \mathbf{g}\,(3,\{2,4\}), \mathbf{c}_{14} + \mathbf{g}\,(4,\{2,3\})\}$$

$$= \mathbf{min}\{1293 + 2013;713 + 2593;821 + 2632\}$$
$$= \mathbf{min} - \{3306;3306;3453\} - = 3306$$

Successor to Node 1 : $p\,(1,\{2,3,4\}) = 3$
Successor to Node 3 : $p\,(3,\{2,4\}) = 4$
Successor to Node 4 : $p\,(4,\{2\}) = 2$
Going back to the optimal TSP path achieves: $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$

The optimal distance is 3306 m (Figs. 7 and 8).

Application of the code with four nodes

Symmetrical distance matrix

First, save the array in a comma-separated CSV file, and then we open the cmd, write the language type (python), then we call the file with the code (app.py). Finally, we take the array file in the specified format (CSV) (Fig. 9; Table 3).

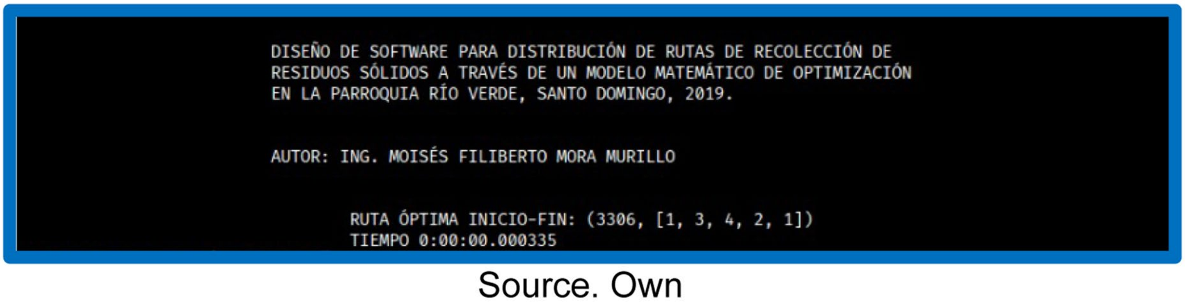


Fig. 9. Console applying the programming code to the example with 4 nodes

Table 3 Example matrix with 4 nodes applying the programming code

	1	2	3	4
1	0	1293	713	821
2	1293	0	1300	619
3	713	1300	0	681
4	821	619	681	0

The optimal route is 1-3-4-2-1, the distance is 3306 m, and the code was calculated in 3.35×10^{-4} s.

Findings

Apply the code in a computer with the following characteristics: Intel CORE i7, sixth generation, RADEON graphics card, 16 GB RAM, 1 TB HDD storage. Applying the code, the mechanical hard disk HDD of the machine collapsed with 18×18 distance matrices, and with lower matrices, it took between 3.00 and 5.00 s to do the computer calculation. The HDD was discontinued and left as a backup storage drive, and we installed a 250GB SSD solid-state hard drive as the main drive of the computer. With the

Table 4 General matrix of routes in the Rio Verde parish

General matrix of routes in the parish of Río Verde in Santo Domingo de los Colorados								
Cooperative, urbanization, citadel, lotization, neighborhood	Graphs	Stage	Place	Optimal start-end route	Optimal distance km	GAS GL.	Price \$. USD.	Code computation time seconds
Ciudadela del Chofer	GRAFO 1-21	1	A	1-9-10-19-21-20-18-17-16-15-14-6-5-4-7-13-12-11-8-2-3-1	1.614	0.058	0.126	98.923
	GRAFO 22-44	2	B	1-18-15-19-21-20-16-13-12-11-9-5-4-8-3-6-17-14-10-7-2-1	3.494	0.126	0.272	88.665
	GRAFO 45-68	3	C	1-13-14-15-16-21-18-20-19-17-12-11-10-6-7-8-9-5-4-3-2-1	1.519	0.055	0.118	87.755
	GRAFO 69-82	4	D	1-13-5-8-9-4-3-10-7-11-12-14-15-6-2-1	1.506	0.054	0.117	0.463
	GRAFO 83-98	5	E	1-3-4-5-6-7-8-11-16-12-13-14-15-9-10-2-1	2.081	0.075	0.162	1.09
Los Girasoles	GRAFO 1-10	1	A	1-10-9-8-7-6-5-4-3-2-1	0.766	0.028	0.06	0.026
	GRAFO 11-19	2	B	1-5-6-7-8-9-4-3-2-1	1.137	0.041	0.088	0.033
	GRAFO 20-29	3	C	1-4-5-9-11-10-6-8-7-3-2-1	1.556	0.056	0.121	0.034
	GRAFO 30-45	4	D	1-3-4-5-7-10-13-12-14-15-16-17-11-8-9-6-2-1	2.159	0.078	0.168	2.389
Patria Nueva	GRAFO 1-21	1	A	1-3-13-14-21-20-19-18-17-16-15-12-11-10-9-8-7-6-5-4-2-1	1.786	0.065	0.139	99.019
	GRAFO 22-30	2	B	1-7-8-6-5-9-4-3-2-1	0.782	0.028	0.061	0.018
Unión Cívica	GRAFO 1-16	1	A	1-8-13-14-15-16-11-12-9-10-5-4-3-6-7-2-1	1.32	0.048	0.103	1.079
	GRAFO 17-31	2	B	1-6-7-8-9-12-14-13-15-11-10-4-3-5-2-1	1.331	0.048	0.104	0.475
	GRAFO 32-47	3	C	1-6-5-7-10-11-14-15-16-13-12-9-8-4-3-2-1	1.251	0.045	0.097	1.122
	GRAFO 48-56	4	D	1-4-7-8-9-6-5-3-2-1	0.793	0.029	0.062	0.023

Table 4 (continued)

General matrix of routes in the parish of Río Verde in Santo Domingo de los Colorados

Cooperative, urbanization, citadel, lotization, neighborhood	Graphs	Stage	Place	Optimal start-end route	Optimal distance km	GAS GL.	Price $. USD.	Code computation time seconds
Montesdeoca	GRAFO 1-17	1	A	1-2-3-6-7-10-11-14-16-17-15-13-12-9-8-5-4-1	0.98	0.035	0.076	2.357
	GRAFO 18-38	2	B	1-3-4-5-8-7-9-10-11-14-15-21-20-19-18-17-16-13-12-6-2-1	3.164	0.114	0.246	89.754
	GRAFO 39-43	3	C	1-5-4-3-2-1	0.953	0.034	0.074	0.019
Cooperativas Coop. Unión Santo D. Cooperativa Pueblo en M. y Cooperativa Defensores P.	GRAFO 1-22	1	A	1-5-4-3-12-13-14 15-16-2-17-18-19-9-20-23-10-11-21-8-7-6-2-1	3.627	0.131	0.282	89.153
Urb. La Castellana	GRAFO 1-21	1	A	1-17-18-20-21-19-16-15-14-13-12-11-10-9-8-7-6-5-4-3-2-1	2.67	0.097	0.208	90.573
	GRAFO 22-42	2	B	1-8-9-12-11-10-6-5-4-15-16-18-17-19-20-21-13-14-2-3-7-1	2.524	0.091	0.196	91.686
	GRAFO 43-46	3	C	1-4-3-2-1	1.192	0.043	0.093	0.015
Coop. Jesús de Gran Poder	GRAFO 1-16	1	A	1-4-7-10-13-14-15-16-12-11-5-6-8-9-3-2-1	1.387	0.05	0.108	0.016
	GRAFO 17-31	2	B	1-4-5-10-7-8-12-13-14-15-11-9-6-3-2-1	1.817	0.066	0.141	0.465
Coop. 16 de Marzo	GRAFO 1-26	1	A	1-5-6-7-9-8-10-11-12-26-25-24-23-22-21-20-19-18-17-13-14-16-15-4-3-2-1	3.527	0.128	0.274	87.296
Coop. Unidos Venceremos	GRAFO 1-28	1	A	1-4-8-9-23-24-20-21-25-28-27-26-22-16-15-14-17-18-13-19-12-7-6-11-10-5-3-2-1	3.35	0.121	0.261	89.815
Coop. Santa Martha sector 1, 2, and 3	GRAFO 1-15	1	A	1-2-5-4-14-15-13-12-11-10-9-8-7-6-3-1	1.771	0.064	0.138	0.467
	GRAFO 16-30	2	B	1-4-14-13-12-11-10-7-6-9-8-5-2-3-1	2.064	0.075	0.161	0.217
Coop. Santa Martha sector 4, 5, and 6	GRAFO 1-16	1	A	1-8-9-16-15-14-13-12-11-6-10-7-5-4-3-2-1	3.516	0.127	0.274	1.161
Coop. Juan Montalvo y Coop. Asistencia Municipal.	GRAFO 1-15	1	A	1-3-4-5-12-15-14-13-9-10-11-6-8-7-2-1	1.414	0.051	0.11	0.519
Coop. Rumiñahui.	GRAFO 1-39	1	A	1-12-13-14-15-16-17-18-19-20-21-9-8-7-6-5-4-3-10-11-2-1	2.307	0.083	0.179	90.462
Coop. Rumiñahui.	GRAFO 22-39	2	B	1-6-7-8-9-16-17-18-15-10-11-12-13-14-4-5-3-2-1	1.755	0.063	0.137	5.543

Table 4 (continued)

General matrix of routes in the parish of Río Verde in Santo Domingo de los Colorados

Cooperative, urbanization, citadel, lotization, neighborhood	Graphs	Stage	Place	Optimal start-end route	Optimal distance km	GAS GL.	Price $. USD.	Code computation time seconds
Coop. Nueva República	GRAFO 1-13	1	A	1-7-6-5-8-12-9-13-10-11-4-3-2-1	1.617	0.058	0.126	0.112
Urb. Los Faisanes	GRAFO 1-12	1	A	1-6-12-7-9-8-11-10-5-4-3-2-1	1.428	0.052	0.111	0.101
Coop. 20 de Octubre	GRAFO 1-11	1	A	1-11-10-9-8-7-6-5-4-3-2-1	1.011	0.037	0.079	0.101
Coop. Juan Montalvo Cosmopolita	GRAFO 1-19	1	A	1-11-12-13-14-15-10-16-19-18-17-9-8-7-4-3-6-5-2-1	2.009	0.073	0.156	15.194
Coop. Santa Martha sector 1	GRAFO 1-23	1	A	1-6-8-9-10-11-12-7-18-23-21-22-20-19-17-15-16-13-14-5-4-3-2-1	4.156	0.15	0.323	93.377
Urb. Skinner y Urb. Ierac 53	GRAFO 1-18	1	A	1-9-11-12-17-16-13-10-14-15-18-8-7-6-5-4-3-2-1	2.93	0.106	0.228	5.479
Urb. Las Palmeras	GRAFO 1-16	1	A	1-12-13-16-15-14-11-10-6-9-8-7-5-4-3-2-1	2.067	0.075	0.161	1.032
Lotización San Ignacio	GRAFO 1-14	1	A	1-5-6-8-7-14-13-12-11-9-10-3-4-2-1	1.088	0.039	0.085	0.215
Barrio San José de Río Verde	GRAFO 1-20	1	A	1-3-5-4-8-9-16-15-10-11-7-6-12-13-14-20-19-18-17-2-1	1.533	0.055	0.119	33.049
Urb. Chanchay	GRAFO 1-13	1	A	1-5-6-13-12-11-10-9-8-7-4-3-2-1	1.391	0.05	0.108	0.099
Coop. Aquepí	GRAFO 1-10	1	A	1-5-7-6-10-9-8-4-3-2-1	1.033	0.037	0.08	0.031
Urb. Los Pambiles	GRAFO 1-12	1	A	1-5-4-7-6-12-11-10-9-8-3-2-1	1.095	0.04	0.085	0.05
Coop. 29 de Diciembre	GRAFO 1-18	1	A	1-7-8-10-9-18-17-16-11-15-14-13-12-6-5-4-3-2-1	1.846	0.067	0.144	5.418
Coop. San Vicente de Paúl	GRAFO 1-15	1	A	1-7-8-13-15-14-12-11-4-3-5-10-9-6-2-1	1.362	0.049	0.106	0.471

Source: Own

250GB SSD, we entered small arrays from 10×10 to 18×18, and the computation time was between 0.30 and 1.00 s. The calculation time for 21 × 21 distance matrices was between 1.00 and 2.00 s (Table 4).

Table 5 shows the results in kilometers, fuel, and cost of the new stable waste collection routes in the Rio Verde parish of Santo Domingo Canton, Santo Domingo de Los Tsa'chilas province.

Table 5 Resources with the new distribution of routes

Optimal distance km	Gas GL.	Price $. USD.	Code computation time seconds
85.679	3.098	6.666	1175.3607

Source: Own

Discussion

At the 16 de Marzo Cooperative, belonging to sector 6 of the Santa Martha Cooperative, the solid waste home collection trucks work on Tuesdays, Thursdays, and Saturdays during the morning hours, so we proceeded to take information by following one of the trucks with the private vehicle throughout our tour in this cooperative (Table 6).

In the 4 weeks, the harvesters travel 6 km or more in the morning working day at the Coop. 16 de Marzo, while the dependent variable such as time is also affected and also exceeds 700 s at 30 km/h to cover the route completely (Table 7).

In hypothesis testing of distance and time variables, we have 24 data on the distance that the collecting truck takes, 12 data without a route, and the other 12 are data on the distance that the truck travels with the new route obtained (Table 8).

Analyzing the 24 sample distance data in the March 16 cooperative, the software (IBM SPSS Statistics 25) yields a significance value of 0.051 with a test value of 4.23 km. With this, the significance value is greater than the percentage error, so we accept the initial hypothesis.

The distance variable is optimized by 30% with the new waste collection route. For the time variable, we have 24 numerical data on the time it takes the collection truck to travel the route, 12 data are conventionally the time it takes the truck to travel as it usually does, and the other 12 data are the time it takes the truck to travel the new route obtained (Table 9).

Analyzing the 24 sample data of the time in the housing cooperative 16 de Marzo, the software (IBM SPSS Statistics 25) gives us a significance value of 0.053 with a test value of 511 s at a speed of 30 km/h. With this, the significance value is higher than the error percentage, so we accept the hypothesis initially stated.

The time variable with the new route for collecting domestic solid waste is 29% more efficient than the conventional route.

Table 6 Field data collection in 16 de Marzo Cooperative

Variables	16 DE Marzo Cooperative, 26 points, 30 km/h, 3.5 km, without new route											
	Week 1			Week 2			Week 3			Week 4		
	Tuesday	Thursday	Saturday	Tuesday	Thursday	Saturday	Tuesday	Thursday	Saturday	Tuesday	Thursday	Saturday
Time (s)	720	732	706.8	708	750	732	744	726	741.6	728.4	710.4	734.4
Distance (km)	6	6.1	5.89	5.9	6.25	6.1	6.2	6.05	6.18	6.07	5.92	6.12

Source: Own

Table 7 Information with the new route of the 16 de Marzo Coop

Variables	16 de Marzo Cooperative, 26 points, 30 km/h, 3.5 km, without new route											
	Week 1			Week 2			Week 3			Week 4		
	Tuesday	Thursday	Saturday	Tuesday	Thursday	Saturday	Tuesday	Thursday	Saturday	Tuesday	Thursday	Saturday
Time(s)	420	420	420	420	420	420	420	420	420	420	420	420
Distance (km)	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5

Source: Own

Table 8 Statistical results to test the hypothesis in the distance variable

Statistics for A sample						
	Test value = 4.23					
	t	gl	Sig. (bilateral)	Difference average	95% confidence interval of the difference	
					Lower	Upper
Total travel distance	2.062	23	0.051	0.5525	−0.0018	1.1068

Source: Own

Table 9 Statistical results to test the hypothesis in the time variable

Statistics for A sample						
	Test value = 511					
	t	gl	Sig. (bilateral)	Difference average	95% confidence interval of the difference	
					lower	upper
Total travel time	2.041	23	0.053	71.23333	−0.9575	143.4242

Source: Own

Conclusions

This code in dynamic programming calculates the optimal routes with matrices of up to 21×21 in only 1 min and 35 s due to the number of operations that the machine must do. The code must calculate the solutions of all the sectors, cooperatives, locations, and settlements in which we have divided the Rio Verde parish. The system finds the solutions it requires for smaller problems using the previous recursive equations and, in turn, taking the answers. The algorithm compares thousands of numbers generated in the computation and discards those that do not comply with minimum values to calculate the optimal distance. We cannot know which sub-problems we need to solve for this type of problem, so we solve them all.

In the San Ignacio location, an optimal route 1-5-6-8-7-14-13-12-11-9-10-3-4-2-1 was obtained in 0.215 s with an asymmetrical distance matrix with 14 points. The truck will cover 1.088 km, investing 0.039 gallons of fuel at the cost of 0.085 dollars.

Starting from the subdivision of sectors of the Rio Verde parish, we have the general data of the code in dynamic programming of the Bellman-Held-Karp method, which we use in a laptop with the following characteristics, Dell, 15 in, RADEON graphics card, Intel i7 of the sixth generation, 1TB of storage, and 16 GB of RAM. Applying the code in this computer gave us routes of 28 sectors in the parish of Rio Verde with 85,679 km in which is invested in 3.098 gallons of diesel at a total cost in Ecuador of US$6.66. To all this, the code took

1175.3607 s to calculate in a computerized way in python language.

Evaluating the Held-Karp model in dynamic programming, we have optimized 29% of the time that a solid waste collector used to take in the time variable. In contrast, in the distance variable, we obtained 30%, very encouraging percentages for future research work in vehicle routing. With the data mentioned above, we accept the hypothesis raised, which establishes that this mathematical model of Held-Karp optimization in dynamic programming provides at least 25% optimization in the study variables, time, and distance in ideal conditions.

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